

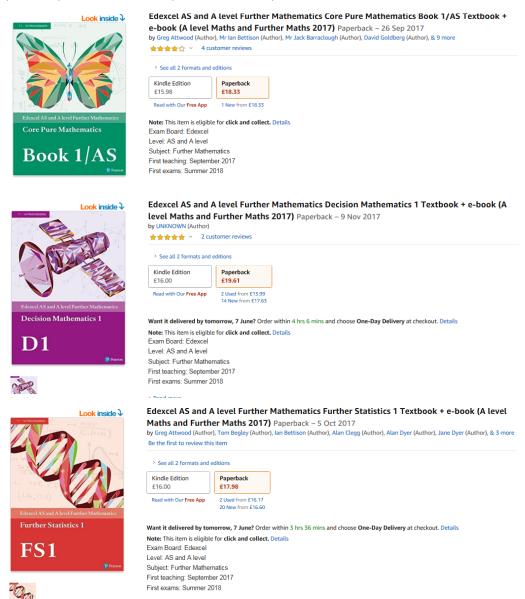
Further Mathematics Bridging Work Booklet 2020

We are excited that you will be joining us to study Further Mathematics in September. We look forward to introducing lots of interesting new topics to you.

There is lots of bridging work for you in the mathematics booklet, but here are just 2 extra topics which will hopefully spark your interest.

Further Mathematics Textbooks

These are the textbooks you need for the course, The Stats and Decision will last for year 12 and year 13, you will need to buy Core book 2 in year 13





Induction Topic 1: Measuring Angle in Radians.

The fact that there are 90 degrees in a right angle will have been familiar to you since primary school, but this number is an arbitrary one which has been passed down to us from the Babylonian civilisation. There was an attempt to introduce 100 degrees in a right angle soon after the French revolution, but this was later dropped and in 1983 a similar attempt was made by the Germans.

But degrees are not the only way to measure angles. The other common measurement for angles is radians, which is much more useful for many applications in mathematics.

If we consider a circle of radius r then the arc length r will subtend an angle of 1 radian at the centre. This means that:

$$180^{\circ} = \pi \text{ radians}$$

And so

degrees	Radians
30°	π/6
45°	π/4
60°	π/3

To convert radians to degrees:	$degrees = radians \times \frac{180}{\pi}$	
To convert degrees to radians:	radians = degrees $\times \frac{\pi}{180}$	
Examples to try:	180	

1. Convert to radians giving your answers to 2 decimal places

Angle in degrees	17	51	243	- 131	- 596
Angle in radians					



2. Convert to radians giving your answer in term of π

Angle in degrees	360	315	- 300	- 630	18
Angle in radians					

3. Convert to degrees giving an exact answer

Angle in degrees					
Angle in radians	$\frac{3\pi}{4}$	$-\frac{7\pi}{6}$	$\frac{2\pi}{5}$	$-\frac{3\pi}{20}$	$\frac{11\pi}{4}$

4. Convert to degrees giving your answer to one decimal place

Angle in degrees					
Angle in radians	0.76	- 1.13	- 11.38	6.42	- 0.032



Degrees and Radians: Solutions

1. Convert to radians giving your answers to 2 decimal places

Angle in degrees	17	51	243	- 131	- 596
Angle in radians	0.30	0.89	4.24	- 2.29	- 10.40

2. Convert to radians giving your answer in term of π

Angle in degrees	360	315	- 300	- 630	18
Angle in radians	2π	$\frac{7\pi}{4}$	$-\frac{5\pi}{3}$	$-\frac{7\pi}{2}$	$\frac{\pi}{10}$

3. Convert to degrees giving an exact answer

Angle in degrees	135	- 210	72	- 27	495
Angle in radians	$\frac{3\pi}{4}$	$-\frac{7\pi}{6}$	$\frac{2\pi}{5}$	$-\frac{3\pi}{20}$	$\frac{11\pi}{4}$

4. Convert to degrees giving your answer to one decimal place

Angle in degrees	43.5	- 64.7	- 652.0	367.8	- 1.83
Angle in radians	0.76	- 1.13	- 11.38	6.42	- 0.032



Induction topic 2: Matrices

A Matrix is a rectangular array of numbers arranged in rows and columns.

The individual numbers in a matrix are called elements.

The order of dimensions of a matrix describes the matrix in terms of the number and rows and columns it has.

Eg
$$\begin{pmatrix} 36 & 21 & 43 \\ 27 & 56 & 35 \end{pmatrix}$$
 has 2 rows and 3 columns so has order 2 x 3

Addition and subtraction of matrices:

Matrices of the same order can be added or subtracted by adding or subtracting the corresponding elements.

Multiplication of a matrix by a number:

Each element of a matrix is multiplied by the multiplying number.

Multiplication of two matrices:

Matrices can be multiplied only if they are compatible. The number of columns in the left hand matrix must be the same as the number of rows in the right hand matrix.

You may find it useful to do an internet search for you tube clips that demonstrate matrix operations.

Questions to try:

3.
$$\mathbf{A} = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$$
 $\mathbf{B} = \begin{pmatrix} -3 & -1 \\ 2 & 7 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 2 & 3 & -4 \\ -1 & 2 & 5 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} -1 & -4 & 2 \\ -3 & 5 & 6 \end{pmatrix}$

Calculate, if possible,

(i)
$$\mathbf{A} + 2\mathbf{B}$$
 (ii) $\mathbf{C} - \mathbf{D}$ (iii) $3\mathbf{A} - 2\mathbf{C}$ (iv) $3\mathbf{D} - \mathbf{C}$

(11)
$$C - I$$

4.
$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$$
 $\mathbf{B} = \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 4 & -1 \\ 2 & 5 \\ -3 & 1 \end{pmatrix}$

Calculate, if possible, the following



Answers

3. (i)
$$A + 2B = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} + 2 \begin{pmatrix} -3 & -1 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} + \begin{pmatrix} -6 & -2 \\ 4 & 14 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -5 \\ 3 & 19 \end{pmatrix}$$

$$(ii) C - D = \begin{pmatrix} 2 & 3 & -4 \\ -1 & 2 & 5 \end{pmatrix} - \begin{pmatrix} -1 & -4 & 2 \\ -3 & 5 & 6 \end{pmatrix}$$
$$= \begin{pmatrix} 3 & 7 & -6 \\ 2 & -3 & -1 \end{pmatrix}$$

(ííí) cannot be done as A and C do not have the same order

(iv)
$$3D-C = 3\begin{pmatrix} -1 & -4 & 2 \ -3 & 5 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 3 & -4 \ -1 & 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -12 & 6 \ -9 & 15 & 18 \end{pmatrix} - \begin{pmatrix} 2 & 3 & -4 \ -1 & 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -15 & 10 \ -8 & 13 & 13 \end{pmatrix}$$

4. (i)
$$AB = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 2 \\ 23 & -5 & -14 \end{pmatrix}$$

(ii)
$$AC = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ -5 & 11 \end{pmatrix}$$

(iii) BC cannot be calculated as the matrices are not conformable (the number of columns in B is not the same as the number of rows in C)

(iv)
$$BD = \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & 5 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -4 & 18 \\ 28 & -2 \end{pmatrix}$$